INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & MANAGEMENT

FUZZY PARAMETERIZED FUZZY SOFT SET FOR MULTIPLE CRITERIA DECISION PROCESS UNDER MULTIPLE EXPERT ASSESSEMENTS

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ABSTRACT
This paper presents aggregate fuzzy set of a fuzzy parameterized fuzzy soft set as model for decision making involving several experts under multiple criteria and an algorithm for aiding the choice of the course of action.

Keywords: fuzzy set, fuzzy parameterized fuzzy soft set, aggregate fuzzy set, multiple criteria.

INTRODUCTION
Decision making is the process of making the right or perhaps choosing the best alternative among competing activities. A desired objective or goal is set subject to certain constraints or limitations and the best alternative cause of action is taken after the systematic analysis of the various alternatives been considered [2]. Several complex problems in real life involving decisions making in the field of mathematics contain uncertainties and methods like the probability theory and interval mathematics have been used to solve these types of problems where classical mathematical approaches are inadequate.
Several research work have emerged to provide solutions to problems of uncertainty such as probability theory, theory of fuzzy sets, rough sets, these theories have their own set back as pointed out by Molodtsov [4]. Molodtsov proposed the idea of soft set theory, a completely new approach for modeling vagueness and uncertainty. However, the approach suffers limitation on parameter set and does not extend to the vagueness and uncertainty of membership of its elements.

Fuzzy set theory was developed by Zadeh [7] to handle situations where there is uncertainty of information or incomplete data. The fuzzy set theory is a concept that establishes that a relationship always exist between certain objects placed in a set [4]. In classical set theory, an element either belongs to a set or it does not belongs to the set in question. However, in fuzzy set an element belongs to a set in degrees which is referred to as the degree of membership within the unit interval [0, 1]. Although there are still issues of inadequacy in the precised evaluation of data containing elements of uncertainty by fuzzy set theory approach. Molodtsov [5] proposed the soft set theory as a new mathematical technique for handling problems where uncertainties exist by introducing parameters.

Cagman et al. [1] hybridized the concepts of fuzzy set theory of Zadeh and the soft set theory of Molodtsov and generated the fuzzy parameterized soft set theory which tries to complement for all the short falls in the individual concepts and its related properties. They defined the empty fuzzy soft set, fuzzy soft subset, the complement of fuzzy soft set and made some propositions. Here, fuzzy soft aggregation operator and also the aggregate fuzzy set also defined. An algorithm for a decision making process and its application is presented.

Maji et al [3] applied the theory of soft sets to solve decision making problem using rough mathematics. An algorithm to select the optimal choice of an object was provided. This algorithm uses fewer parameters to select the optimal object amongst alternatives for decision problem. The decision making problem had a straight forward relationship between the decision values and the conditional parameters.

Xiao et al. [6] studied synthetically evaluating method for business competitive capacity based on soft set. Their approach overcomes the deficiency of traditional methods of dealing with uncertainties by bringing an algorithm that is very convenient and easily applicable in practice.
Kalaichelvi and Malini [2] constructed a model based on fuzzy soft set theory for decision making problem, they considered parameters like safety of funds, liquidity of funds, high returns, tax concession, easy accessibility, stable

return and the model proved successful and they concluded that the concept of fuzzy soft set has a rich possibility for developing decision making models suitable for a personal, commercial and managerial issues.
In this paper we present fuzzy parameterized fuzzy soft set as a model for decision making process involving several experts under multiple criteria with the aid of a proposed algorithm. Consequently, we present some basic definitions on fuzzy set, fuzzy parameterized fuzzy soft set and aggregate fuzzy set in section 2. In section 3, we propose an algorithm for aiding decision making process in the choice of the best option among alternatives offered by experts under multiple criteria assessement parameters. Here an example of a decision making problem is presented and tested consisting of assessement of a student's score in a final year project defense by ten lecturers (experts) under seven criteria. In section 4 we summarize our findings with recommendation and further research directions.

BASIC DEFINITIONS

Definition 2.1.1 Let U be a universal set . A fuzzy set A over U is a set defined by a function

$$\mu_A: U \rightarrow [0, 1]$$

Here, μ_A called membership function of A, and the value $\mu_A(x)$ is called the grade of membership of $x \epsilon U$. The value represents the degree of x belonging to the fuzzy set. Thus; a fuzzy set A over U can be represented as follows

$$\mathrm{A} = \{(\mu_A(x) / x): x \epsilon U, \mu_A(x) \epsilon [0,1]\}$$

We shall use $F(U)$ to denote the set of fuzzy sets over U .

Definition 2.1.2 The support of a fuzzy set A over a universe U is the set containing all the elements that have no-zero membership grades in A denoted by $\mathrm{Supp}(A)$ that is:

$$\mathrm{Supp}(A) = \{x \epsilon U | \mu_A(x) > 0\}$$

Definition 2.1.3 [ ] Let U be a universal set, E be the set of all parameters and A be a fuzzy set over E with membership function $\mu_A: E \rightarrow [0,1]$ and $\gamma_A(x)$ be a fuzzy set over U for all $x \in E$. Then a fuzzy parameterized fuzzy soft set ($fpfs$_set) Γ_A over U is a set defined by the function $\gamma_A(x)$ representing a mapping:

$$\gamma_A: E \rightarrow F(U) \text{ such that } \gamma_A(x) = \emptyset \text{ if } \mu_A(x) = 0.$$

Here, γ_A is called fuzzy approximate function of the _set Γ_A. The value $\gamma_A(x)$ is a fuzzy set called x-element of the $fpfs$_set for all $x \in E$. Thus, an $fpfs$_set Γ_A over U can be represented by the set of ordered pairs:

$$\Gamma_A = \{(\mu_A(x)/(x), \gamma_A(x)): x \epsilon E, \gamma_A(x) \epsilon F(U), \mu_A(x) \epsilon [0,1]\}.$$

The sets of all fuzzy parameterized fuzzy soft sets over U is denoted by $FPFS(U)$.

Example 1. Assuming $U = \{u_1, u_2, u_3, u_4\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of parameters. If $A = \{0.3/x_2, 0.7/x_3, 0.8/x_4\}$ and
$\gamma_A(x_1) = \emptyset, \gamma_A(x_2) = \{0.5/u_2, 0.1/u_3\}, \gamma_A(x_3) = \{0.3/u_1, 1/u_2\}, \gamma_A(x_4) = U,$
$\gamma_A(x_5) = \emptyset$. Then $fpfs$_set Γ_A is written:

$$\Gamma_A = \{(0.3/x_2, \{0.5/u_2, 0.1/u_3\}), (0.7/x_3, \{0.3/u_1, 1/u_2\}), (0.8/x_4, U)\}$$

Definition 2.1.4 [1] The $fpfs$_aggregation operator, denoted by $FPFS_{agg}$, is defined:

$FPFS_{agg}: F(E) \times FPFS(U) \rightarrow F(U)$ such that $FPFS_{agg}(A, \Gamma_A) = \Gamma_A^*$ where

$\Gamma_A^* = \left\{\mu_{\Gamma_A^{*}(u)}/u \,\middle|\, u \in U\right\}$ which is a fuzzy set over U. The value Γ_A^* is called aggregate fuzzy set of the Γ_A. Here the membership function $\mu_{\Gamma_A^{*}(u)}$ of u is defined:

$\mu_{\Gamma_A^{*}(u)} = \frac{1}{|E|}\sum \mu_A(x)\mu_{\gamma_A(x)}(u)$

where $|E|$ is the cardinality of E.

Example 2. From example 1, we have
$|E| = 5$, $\mu_{\Gamma_A^{*}(u_1)} = \frac{1}{5}(0.7 \times 0.3 + 0.8 \times 1) = 0.202$
$\mu_{\Gamma_A^{*}(u_2)} = \frac{1}{5}(0.3 \times 0.5 + 0.7 + 0.8) = 0.33$
$\mu_{\Gamma_A^{*}(u_3)} = \frac{1}{5}(0.3 \times 0.1 + 0.8) = 0.17$
$\mu_{\Gamma_A^{*}(u_4)} = \frac{1}{5}(0.8 \times 1) = 0.16$
Therefore, $\Gamma_A^* = \{0.202/u_1, 0.33/u_2, 0.17/u_3, 0.16/u_4\}$.

fpfs-SET AS A MODEL FOR MULTIPLE CRITERIA UNDER MULTIPLE EXPERTS' ASSESSEMENT DECISION PROCESS

Let obj denote an object to be evaluated under a set of n qualitative or quantitative criteria $E = \{x_1, x_2, \dots, x_n\}$ by a set of m expert assessors $U = \{u_1, u_2, \dots, u_m\}$, we propose the following algorithm for the choice of an assessement score:

Step 1. Generate the fuzzy set $A = \left\{\mu_A(x_j)/x_j \,\middle|\, \mu_A(x_j) = \frac{E(x_j)}{\sum_{j=1}^{n} E(x_j)}\right\}$ where

$E(x_i)$ is the expected score on the parameter x_i by obj.

Step 2. Generate the class of fuzzy sets $F(U) = \left\{\gamma_A(x_j) \,\middle|\, \mu_{\gamma_A(x_j)}(u_i) = \frac{a_{ij}}{E(x_j)}\right\}$ where

where a_{ij} is the assessement score of obj by assessor u_i on the criteria x_j

Step 3. Construct an *fpfs*-set Γ_A over U.
Step 4 . Find the aggregate fuzzy set Γ_A^* of Γ_A
Step 5. Find the largest membership grade $max\ \mu_{\Gamma_A^{*}(u)}$ and
Step 5. Select u for which $\alpha = max\ \mu_{\Gamma_A^{*}(u)}$ or else,
Step 6. If $max\ \mu_{\Gamma_A^{*}(u)} = \alpha_{i_1}, \alpha_{i_2}, \dots, \alpha_{i_r}$ then select u_i such that $\mu_{\Gamma_A^{*}(u_i)} = \alpha_{i_k}, k = 1,2,\dots,r$
Step 7. Compute and select $\bar{u} = \frac{\sum_{i=i_1}^{i_r}\sum_{j=1}^{n} a_{ij}}{i_r}$

Example 3. The scores of one student in project internal defense computer science as graded by 10 committee of experts is presented in table 1 below with the set of parameters
$E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ where
x_1 = Formulation and defining of study problem and objectives; theoretical and conceptual framework. (6 marks)
x_2 = Use of relevant literature, secondary sources of data and reference. (3 marks)
x_3 = Data collection, methodology; Adequacy and relevance of data collected. (6 marks)
x_4 = Data handling and analysis, illustrations, appropriateness, and relevance to the study. (6 marks)
x_5 = Interpretation and logical presentation of information, critical discussion, literature support and conclusion. (3 marks)

x_6 = Adequacy, Accuracy of information presented: usefulness of the study. (3 marks)
x_7 = English expression, language clarity and communications. (3 marks)
U = Committee members : $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$

	x_1 6	x_2 3	x_3 6	x_4 6	x_5 3	x_6 3	x_7 3
u_1	4	2	3	3	2	2	2
u_2	4	1	3	3	2	2	2
u_3	3	2	3	3	2	2	2
u_4	3	2	3	3	2	1	1
u_5	4	2	3	3	2	1	2
u_6	4	2	3	3	2	1	2
u_7	4	2	4	3	2	2	2
u_8	4	2	3	3	2	1	2
u_9	2	2	2	2	1	1	1
u_{10}	4	2	3	2	2	1	2

Table 1: Scores of a final year student under ten expert assessements in project internal defense for B.Sc (Hons) computer science.

For all $i = 1,2,3,4,5,6,7$, we have $E(x_1) = 6$, $E(x_2) = 3$, $E(x_3) = 6$, $E(x_4) = 6$, $E(x_5) = 3$, $E(x_6) = 3$, $E(x_7) = 3$ and $\mu_A(x_j) = \frac{E(x_j)}{\sum_{i=1}^{n} E(x_i)}$

We denote the assessed score of a student by expert u_i on parameter x_j by a_{ij} so that $\mu_{\gamma_A(x_j)}(u_i) = \frac{a_{ij}}{E(x_j)}$

From table 1, we have the following table expressed in terms of the functions μ_A and $\mu_{\gamma_A(x_j)}$.

μ_{γ_A}	$\boldsymbol{x_1}$ **0.2**	$\boldsymbol{x_2}$ **0.1**	$\boldsymbol{x_3}$ **0.2**	$\boldsymbol{x_4}$ **0.2**	$\boldsymbol{x_5}$ **0.1**	$\boldsymbol{x_6}$ **0.1**	$\boldsymbol{x_7}$ **0.1**
$\boldsymbol{u_1}$	0.7	0.7	0.5	0.5	0.7	0.7	0.7
$\boldsymbol{u_2}$	0.7	0.3	0.5	0.5	0.7	0.7	0.7
$\boldsymbol{u_3}$	0.5	0.7	0.5	0.5	0.7	0.7	0.7
$\boldsymbol{u_4}$	0.5	0.7	0.5	0.5	0.7	0.3	0.3
$\boldsymbol{u_5}$	0.7	0.7	0.5	0.5	0.7	0.3	0.7
$\boldsymbol{u_6}$	0.7	0.7	0.5	0.5	0.7	0.3	0.7
$\boldsymbol{u_7}$	0.7	0.7	0.7	0.5	0.7	0.7	0.7
$\boldsymbol{u_8}$	0.7	0.7	0.5	0.5	0.7	0.3	0.7
$\boldsymbol{u_9}$	0.3	0.7	0.3	0.3	0.3	0.3	0.3
$\boldsymbol{u_{10}}$	0.7	0.7	0.5	0.3	0.7	0.3	0.7

Table 2 showing μ_A and μ_{γ_A} functions

From table 2 above we have:
$A = \{0.2/x_1, 0.1/x_2, 0.2/x_3, 0.2/x_4, 0.1/x_5, 0.1/x_6, 0.1/x_7\}$

$$\Gamma_A = \begin{Bmatrix} (0.2/x_1, \{0.7/u_1, 0.7/u_2, 0.5/u_3, 0.5/u_4, 0.7/u_5, 0.7/u_6, 0.7/u_7, 0.7/u_8, 0.3/u_9, 0.7/u_{10}\}), \\ (0.1/x_2, \{0.7/u_1, 0.3/u_2, 0.7/u_3, 0.7/u_4, 0.7/u_5, 0.7/u_6, 0.7/u_7, 0.7/u_8, 0.7/u_9, 0.7/u_{10}\}), \\ (0.2/x_3, \{0.5/u_1, 0.5/u_2, 0.5/u_3, 0.5/u_4, 0.5/u_5, 0.5/u_6, 0.7/u_7, 0.5/u_8, 0.3/u_9, 0.5/u_{10}\}), \\ (0.2/x_4, \{0.5/u_1, 0.5/u_2, 0.5/u_3, 0.5/u_4, 0.5/u_5, 0.5/u_6, 0.5/u_7, 0.5/u_8, 0.3/u_9, 0.3/u_{10}\}), \\ (0.1/x_5, \{0.7/u_1, 0.7/u_2, 0.7/u_3, 0.7/u_4, 0.7/u_5, 0.7/u_6, 0.7/u_7, 0.7/u_8, 0.3/u_9, 0.7/u_{10}\}), \\ (0.1/x_6, \{0.7/u_1, 0.7/u_2, 0.7/u_3, 0.3/u_4, 0.3/u_5, 0.3/u_6, 0.7/u_7, 0.3/u_8, 0.3/u_9, 0.3/u_{10}\}), \\ (0.1/x_7, \{0.7/u_1, 0.7/u_2, 0.7/u_3, 0.3/u_4, 0.7/u_5, 0.7/u_6, 0.7/u_7, 0.7/u_8, 0.3/u_9, 0.7/u_{10}\}) \end{Bmatrix}$$

So that

$$\mu_{\Gamma_A^*(u_1)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.7 + 0.1 \times 0.7 \end{pmatrix} = 0.088571.$$

$$\mu_{\Gamma_A^*(u_2)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.3 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.7 + 0.1 \times 0.7 \end{pmatrix} = 0.0828571.$$

$$\mu_{\Gamma_A^*(u_3)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.5 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.7 + 0.1 \times 0.7 \end{pmatrix} = 0.0828571.$$

$$\mu_{\Gamma_A^*(u_4)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.5 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.3 + 0.1 \times 0.3 \end{pmatrix} = 0.0714286.$$

$$\mu_{\Gamma_A^*(u_5)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.3 + 0.1 \times 0.7 \end{pmatrix} = 0.0828571.$$

$$\mu_{\Gamma_A^*(u_6)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.3 + 0.1 \times 0.7 \end{pmatrix} = 0.0828571..$$

$$\mu_{\Gamma_A^*(u_7)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.7 + 0.2 \times 0.7 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.7 + 0.1 \times 0.7 \end{pmatrix} = 0.0942857.$$

$$\mu_{\Gamma_A^*(u_8)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.5 + 0.1 \times 0.7 + 0.1 \times 0.3 + 0.1 \times 0.7 \end{pmatrix} = 0.0828571.$$

$$\mu_{\Gamma_A^*(u_9)} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.3 + 0.1 \times 0.7 + 0.2 \times 0.3 \\ +0.2 \times 0.3 + 0.1 \times 0.3 + 0.1 \times 0.3 + 0.1 \times 0.3 \end{pmatrix} = 0.0485714.$$

$$\mu_{\Gamma_A^*(u_{10})} = \frac{1}{7}\begin{pmatrix} 0.2 \times 0.7 + 0.1 \times 0.7 + 0.2 \times 0.5 \\ +0.2 \times 0.3 + 0.1 \times 0.7 + 0.1 \times 0.3 + 0.1 \times 0.7 \end{pmatrix} = 0.0771429.$$

Therefore, $\Gamma_A^* = \begin{Bmatrix} 0.088571/u_1, 0.0828571/u_2, 0.0828571/u_3, 0.0714286/u_4, 0.0828571/u_5 \\ 0.0828571/u_6, 0.0942857/u_7, 0.0828571/u_8, 0.0485714/u_9, 0.0771429/u_{10} \end{Bmatrix}$
and $max\ \mu_{\Gamma_A^*(u)} = 0.0942857$ on assessement u_7 . Consequently, the assessement scores by the expert u_7 are selected for obj.i.e the total score $4 + 2 + 4 + 3 + 2 + 2 + 2 = 19$.

CONCLUSION

In this paper, the concept of fuzzy parameterized fuzzy soft set has been proposed as a decision tool for a multiple criteria decision process under the assessement of several experts. An algorithm aiding the choice of an alternative course of action also proposed with a practical demonstration on students' final year internal project assessements by several experts. Since average method is influenced by extreme values and may not be a true representation of a reality, our method minimizes these disadvantages. However, the efficiency of our method needs to be investigated.

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